MODIFIED COMPRESSION FIELD THEORY (MCFT) FOR SHEAR STRENGTH PREDICTIONS OF REINFORCED MASONRY SHEAR WALLS

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Reinforced masonry (RM) shear walls detailed for use as a Seismic Force Resisting System (SFRS) in Canada are subject to overly conservative methods of determining their shear strength that may result in uneconomic and inefficient designs. In the current study, the results of eight full-scale squat RM walls tested at McMaster University to quantify their shear strength, were used to investigate the application of the Modified Compression Field Theory (MCFT), originally developed for reinforced concrete (RC) components, to predict their shear strength. The test walls demonstrated shear-strength capacities up to 200% of those predicted by the Canadian Standards Association (CSA) S304.1-04 masonry design code. However, the *Simplified Modified Compression Field Theory* (SMCFT), adopted directly from the Canadian concrete design code CSA A23.3-04, showed promise for future applications with RM masonry walls as it resulted in more accurate estimate of shear strength compared to the current S304.1-04 masonry design code approach.

Keywords: *Reinforced masonry, Seismic performance, Shear failure, Shear walls*

INTRODUCTION

The behaviour of reinforced masonry (RM) walls failing in shear is complex because of the limited information on shear compared to flexural behaviour, the anisotropic nature of masonry and the nonlinear composite material interaction (e.g. masonry units, mortar, grout, vertical steel and horizontal steel) within a typical RM shear wall. Different researchers have proposed a number of masonry shear strength expressions (Matsumura, 1988, Shing et al., 1990, Anderson and Priestley, 1992 and Voon and Ingham, 2007) based on their own experimental results or on surveys of experimental tests reported in literature. These expressions are typically presented into a linear algebraic summation of the contributions of shear force resisting mechanisms within a RM wall in an effort to simplify the design process. However, shear design expressions of RM walls vary by jurisdiction in their formulation and contain different prescriptive design requirements and calibration factors due to the variability in construction practices and seismic hazard. In addition, comparison between international design codes is made complicated by material-reduction and load-amplification factors (within limit state design) that are often applied separately from the basic shear strength formulation but are still calibrated with the seismic hazard level or construction quality within their respective jurisdictions.

CURRENT CANADIAN MASONRY DESIGN CODE APPROACH

The current Canadian shear design expression given in the CSA S304.1-04 (CSA 2004a) is based on an assumed shear resistance plane acting at a 45° angle across an effective wall depth (d_v) and is comprised of three superimposed resisting mechanisms: masonry shear strength, shear strength enhancement due to applied axial load and shear reinforcement strength. The CSA S304.1-04 expression for in-plane shear resistance (V_n) of a RM shear wall is given by Eq. 1.

$$V_n = (v_m b_w d_v + 0.25P_d)\gamma_g + \left(0.6A_h f_y \frac{d_v}{s_h}\right) \tag{1}$$

where $v_m = 0.16\left(2 - \frac{h_w}{d_v}\right)\sqrt{f'_m}$ (2)

In Eq.1, both the masonry and steel material strength reduction factors, ϕ_m=0.6 and ϕ_s=0.85, are omitted in order to facilitate comparison with the experimental data and the MCFT concrete code expressions as will be shown later. The shear resistance provided by the masonry units and axial force is described by γ_g as the factor accounting for partial grouting in the wall (γ_g =1.0 for fully-grouted RM), v_m as the masonry shear strength given in Eq. 2 in MPa, b_w as the thickness of the wall in mm, d_v as the effective depth of the wall which need not be taken as less than 80% the wall length (ℓ_w) in mm and P_d as the axial compressive dead load in N. The resistance provided by the shear reinforcement is described by A_h as the cross-sectional area of shear reinforcement in mm^2, f_y as the yield strength of shear reinforcement in MPa and s_h as the vertical spacing of shear reinforcement.

CURRENT CANADIAN CONCRETE DESIGN CODE APPROACH

The *General Method* shear design expression given by the CSA A23.3-04 (CSA, 2004b) for design of RC walls, was selected for comparison. Although the latter expression has not been derived accounting for the anisotropic nature of masonry, it was nevertheless selected due to the general similarity in the behavior of RC and fully-grouted RM shear walls. This shear strength expression was derived from the Simplified Modified Compression Field Theory (SMCFT) for RC elements as described by Bentz et al. (2006) and as presented in Eq. 3. It is worth noting that more recently, Sarhat and Sherwood (2010) observed also that this expression was more accurate than the expression in CSA S304.1-04 (CSA 2004a) for predicting the shear strength of RM beams.

$$V_n = \beta\sqrt{f'_m}\, b_w d_v + \frac{A_h f_y d_v \cot\theta}{s_h} \tag{3}$$

where $\beta = \frac{0.40}{(1+1500\varepsilon_x)} \times \frac{1300}{(1000+s_{ze})}$; (4)

$\theta = 29 + 7000\varepsilon_x$; (5)

and $\varepsilon_x = \frac{V_n h_w / d_v + V_n - 0.5P_d}{2E_s A_v}$ (6)

The shear strength of Eq. 3 is a summation of the two resistance mechanisms attributed to concrete and steel reinforcement. This expression accounts for the resistance offered along shear cracks that form at an angle (θ) that may deviate from the 45° angle assumed by masonry expressions. Utilizing the SMCFT expression, the resistance offered by the masonry carried across a crack would be described by the constant β, where $s_{ze} = 35s_z / (10 + a_g)$ and s_z is a crack spacing parameter, taken as the vertical reinforcement spacing of 200 mm in the tested RM walls, and a_g is the nominal maximum size of course aggregate, taken as 10 mm as suggested by Sarhat and Sherwood (2010) for coarse masonry grout. The longitudinal strain at mid-depth of the RM wall (ε_x) would then be solved for iteratively by initially guessing a value of V_n and solving Eq. 6, a solution will converge when V_n used in Eq. 6 becomes equal to V_n determined by Eq. 3. The negative sign in front of P_d accounts for a compressive axial force (positive for tension) (CSA, 2004b).

The CSA S304.1-04 (CSA 2004a) shear strength expression (Eq. 1) employs empirically derived reduction factors to the resistance of shear reinforcement (given as 0.6 in Eq. 1). A modified shear expression is given in Eq. 7 as an alternative to that of the current CSA S304.1-04 (CSA 2004a) where the 0.6 reduction factor has been removed. This expression will be referenced hereafter as "Modified CSA S304.1" as given by:

$$V_n = (v_m b_w \ell_w + 0.25P_d)\gamma_g + \left(A_h f_y \frac{d_v}{s} \right) \qquad (7)$$

TEST MATRIX USED FOR COMPARISON

The RM shear wall selected for code evaluation were reported by Miller et al. (2005). The walls have been detailed with a range of design parameters corresponding to those found in typical low-rise RM construction including masonry unit compressive strength (f'_m), wall height-to-length (aspect) ratio ($A_r=h_w/\ell_w$), flexural reinforcement ratio (ρ_v), horizontal reinforcement spacing (s_h) and the level of applied axial stress (σ_n). The test walls have been specifically detailed with relatively high flexural reinforcement ratio in order to ensure the development of a shear mechanism and hence, minimizing flexural reinforcement yielding so that any realized ductility will be attributed to a shear damage mechanism. The construction details for each test wall are presented in Table 1.

Table 1: Wall Design Details

	Masonry Strength	Wall Dimensions		Aspect Ratio	Vertical Reinforcement			Horizontal Reinforcement			Axial Load
Wall	f'_m (MPa)	ℓ_w (m)	h_w (m)	A_r	A_v (mm^2)	s_v (mm)	ρ_v (%)	A_h (mm^2)	s_h (mm)	ρ_h (%)	σ_n (MPa)
W-1	15.4	2.0	2.0	1.0	300	200	0.79	100	800	0.079	1.0
W-2	12.7	2.0	2.0	1.0	300		0.79		400	0.13	0
W-3	15.4	2.0	3.0	1.5	500		1.32		400	0.12	0
W-4	12.7	3.0	2.0	0.67	500		1.32		400	0.13	1.0
W-5	12.7	2.0	3.0	1.5	500		1.32		800	0.070	1.0
W-6	15.4	3.0	3.0	1.0	300		0.79		400	0.12	1.0
W-7	15.4	3.0	2.0	0.67	500		1.32		800	0.079	0
W-8	12.7	3.0	3.0	1.0	300		0.79		800	0.070	0

COMPARISON BETWEEN Code predictions

Since all walls were tested with reversed cycles of loading, code shear strengths are compared to the averaged value of Q_{ult} from both directions of loading. To facilitate comparison, the average measured experimental results are normalized by the code predictions and presented as the ratio Q_{ult}/V_n for each shear strength expression in Table 2.

Table 2. Normalized shear strength Q_{ult}/V_n using the three Shear Strength Expressions

Wall	Q_{ult}/V_n		
	Current CSA S304.1	**Modified CSA S304.1**	**CSA A23.3 General Method**
W-1	1.21	1.10	1.04
W-2	1.61	1.29	1.00
W-3	1.49	1.21	0.89
W-4	1.61	1.38	1.18
W-5	1.21	1.09	0.98
W-6	1.30	1.10	1.05
W-7	2.02	1.80	1.26
W-8	1.63	1.41	1.04
Mean	**1.51**	**1.30**	**1.06**
c.o.v.	**18.0%**	**18.3%**	**10.8%**

As can be inferred from Table 2, the most accurate prediction of shear strength was *The General Method* based on the CSA A23.3-04 (CSA 2004b) concrete design approach. Although not originally developed for use with RM, it yielded the most accurate mean of Q_{ult}/V_n = 1.06 and the lowest coefficient of variation (c.o.v.) of 10.8%. The modifications to the CSA S304.1-04 equation (CSA 2004a) given by the Modified CSA S304.1 expression (Eq. 13) in Table 2 resulted in some improvement in terms of the mean Q_{ult}/V_n = 1.30 (from 1.51) with a c.o.v. of 18.3% (from 18.0%) compared to the current CSA S304.1-04 (CSA, 2004a) expression.

LOAD-SHEAR DISPLACEMENT RELATIONSHIPS

Analyses of flexure-critical RM shear walls lead to an explicit evaluation of the displacement ductility utilizing beam theory based on the first occurrence of yield strain in the flexural reinforcement and the subsequent development of wall cross-section curvature ductility within a plastic hinge region. However, the complexity of shear-critical RM wall behaviour and damage mechanism, that has arguably led to the development of overly conservative and empirical strength expressions, has also complicated the development of simplified and accurate means to generate load-shear displacement relationships. In this regard, the development of the latter presents two problems, namely: determining the effective *shear yield displacement* and estimating the shear-critical wall *ultimate displacement*. This is attributed to the fact that, even with the subtraction of base sliding, the top wall displacement is a summation of interdependent shear and flexural deformations that, although maybe decoupled from experimental measurements, are complex to predict or model for RM walls.

Englekirk (2003) suggests proposed a simplified model to predict the shear yield displacement of RC beams as a function of the beam geometry and the yield strain of the shear reinforcement. In his model, the shear yield displacement is estimated by developing a strut-and-tie model with the strut acting at 45° and the shear reinforcement acting as ties. As such, a RC cantilever is discretized into square panels contributing to the overall shear yield displacement. The overall shear yield displacement would then be the summation of the panel displacements, which, for a squat RM wall, can be shown to be given by Eq. 8.

$$\Delta_{y,shear} = 1.25\varepsilon_{sy} d_v \leq 1.25\varepsilon_{sy} h_w \quad (8)$$

The yield strain of the horizontal reinforcement (ε_{sy}) is taken as 0.002125 as per the steel test data and the corresponding shear yield displacement for each wall based on Eq. 8 is presented in Fig. 1 as the vertical dashed line referring to Englekirk's (2003) model.

Vecchio and Collins (1986) and later Bentz et al. (2006) asserted that the crack inclination will change over the loading history of a RC element under shear. Vecchio and Collins (1986) also indicated that the point where shear reinforcement yields does not correlate well with the elastic-perfectly plastic bilinear relationship commonly adopted for flexural ductility determination. As such, each of the tested RM walls has been idealized as a membrane element subjected to pure shear, the load-shear drift relationship has been determined with the MCFT under a state of constant strain and neglecting flexural deformations.

The resulting load-shear drift relationships are presented in Fig. 1 for each wall specimen along with the experimental load- shear drift envelope extracted from the experimental data. In addition, the shear drift, at which the MCFT analysis predicts initial yielding of shear reinforcement, is identified by a vertical line in Fig. 1.

The predicted ultimate displacement of each wall was evaluated at the point of crushing of the compression strut in each wall and is indicated in Fig. 1 by the vertical dotted line. The results presented in Fig. 1 indicate some success towards adopting the MCFT as a simplified approach to estimating the wall load-shear deformation relationships. Walls W-1, W-2 and W-6, with an aspect ratio of 1.0, had reasonable resemblance of the experimentally-derived load-shear drift envelope to that based on the MCFT approach with failure governed by crushing of the compression strut. However, neither Eq. 8 (Englekirk's model) nor the MCFT simplification accurately estimated the shear yield displacement of the load-shear drift envelope. In this regard, recent experimental testing of RC piers and spandrels by Massone et al. (2009) concluded that transverse horizontal strains were inhibited by the relative fixity provided by the top and bottom wall connections.

In addition, Beyer et al. (2011) also noted that shear deformations tend to concentrate in regions of walls where curvatures are high. Therefore, the simultaneous yielding of horizontal reinforcement over the entire wall height is unlikely to occur, as assumed in Eq. 8 and the MCFT model, but more likely to occur in the mid-to-lower region of a wall where curvatures are maximized. In conclusion, Fig. 1 indicates that estimating the shear yield displacement of squat RM walls based on the first yield of shear reinforcement is overly conservative for the tested walls. Furthermore, applying MCFT to estimate shear deformations showed a very good fit with Wall W-2 since it most closely resembled an element under pure shear.

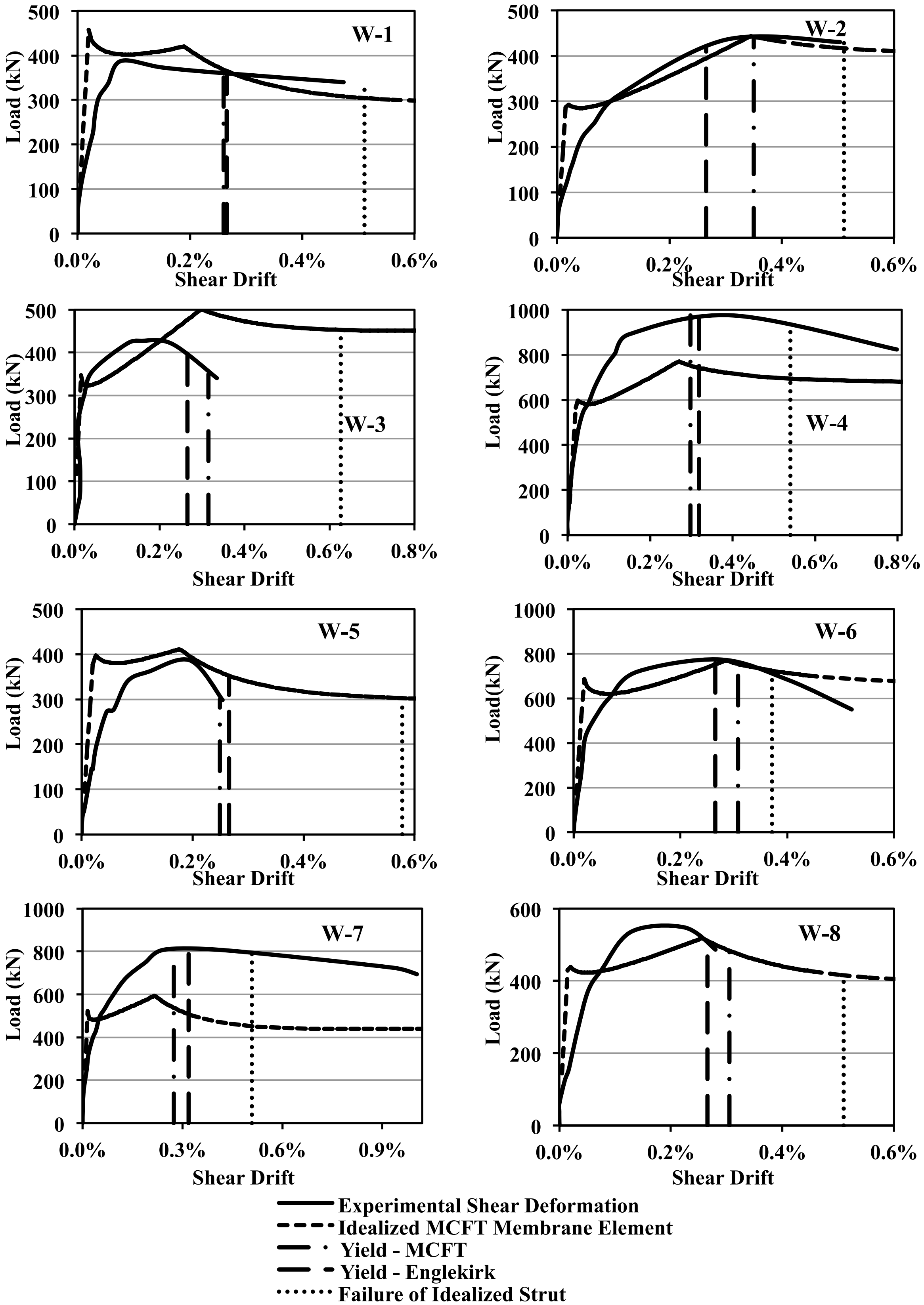

Figure 1: Lateral Resistance Versus Average Shear Deformation of Walls

CONCLUSIONS

Based on the paper analysis, it appears that the current CSA S304.1-04 (CSA, 2004a) shear strength expression produced very conservative predictions (Q_{ult}/V_n = 1.51, c.o.v. = 18.0%). On the other hand, the Simplified Modified Compression Field Theory (SMCFT) adopted in design of RC members in the CSA A23.3-04 (CSA, 2004b) yielded very accurate predictions of the tested RM wall shear strength with a mean Q_{ult}/V_n = 1.06 and a low c.o.v. of 10.8%. Adopting the SMCFT appears also promising in terms of predicting RM wall force-shear displacement relationships for walls essentially failing in shear without flexural hinging. The SMCFT offers a relatively simple approach to estimating the peak shear strength of RM walls and with further refinement over a broader range of wall tests and accounting for masonry-specific behavior have the potential of being adopted in masonry design codes.

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